

# Testing the locality of transport in self-gravitating accretion discs

Giuseppe Lodato<sup>\*</sup> and W. K. M. Rice<sup>†</sup>

<sup>\*</sup>*Institute of Astronomy, Cambridge, UK*

<sup>†</sup>*School of Physics and Astronomy, University of St. Andrews, St. Andrews, UK*

**Abstract.** In this paper we examine the issue of characterizing the transport associated with gravitational instabilities in relatively cold discs, discussing in particular under which condition it can be described within a local, viscous framework. We present the results of global, three-dimensional, SPH simulations of self-gravitating accretion discs, in which the disc is cooled using a simple parameterization for the cooling function. Our simulations show that the disc settles in a “self-regulated” state, where the axisymmetric stability parameter  $Q \approx 1$  and where transport and energy dissipation are dominated by self-gravity. We have computed the gravitational stress tensor and compared our results with expectations based on a local theory of transport. We find that, for disc masses smaller than  $0.25M_*$  and aspect ratio  $H/r < 0.1$ , transport is determined locally, thus allowing for a viscous treatment of the disc evolution.

## INTRODUCTION

One of the basic unknowns in accretion disc theory is the physical mechanism ultimately responsible for angular momentum transport and energy dissipation in the disc. The usual way to overcome this difficulty is to assume that transport is dominated by some kind of viscous process, and to give an *ad hoc* prescription for the  $r\phi$  component of the stress tensor  $T$  (usually vertically integrated, under the assumption that the disc is geometrically thin). The most widely used prescription is the so-called  $\alpha$ -prescription [1], according to which:

$$T_{r\phi} = \left| \frac{d \ln \Omega}{d \ln r} \right| \alpha \Sigma c_s^2, \quad (1)$$

where  $\Omega$  is the angular velocity of the disc,  $c_s$  is the thermal speed,  $\Sigma$  is the surface density, and  $\alpha$  a free parameter, expected to be smaller than unity.

It has recently been recognized that accretion discs threaded by a weak magnetic field are subject to MHD instabilities (see Balbus and Hawley [2] and references therein), that can induce turbulence in the disc, thereby being able to transport angular momentum outwards and to promote the accretion process. However, in many astrophysically interesting cases, such as the outer regions of protostellar discs, the ionization level is expected to be very low, reducing significantly the effects of magnetic fields in determining the dynamics of the disc. A possible alternative source of transport in cold discs is provided by gravitational instabilities.

The axisymmetric stability of a thin disc with respect to gravitational disturbances is determined by the condition [3]:

$$Q = \frac{c_s \kappa}{\pi G \Sigma} > \bar{Q} \approx 1, \quad (2)$$

where  $\kappa$  is the epicyclic frequency. The external regions of many observed systems are likely to be gravitationally unstable. If we consider, for example, the case of AGN accretion discs, it can be shown [4] that  $Q$  falls below unity already at a distance of  $\approx 10^{-2}$  pc from the central black hole. In the case of protostellar discs, models of the outburst in FU Orionis systems (a class of young stellar objects that experience a phase of enhanced accretion) show that the disc is marginally stable already at a distance of  $\approx 1$  AU from the central young star [5].

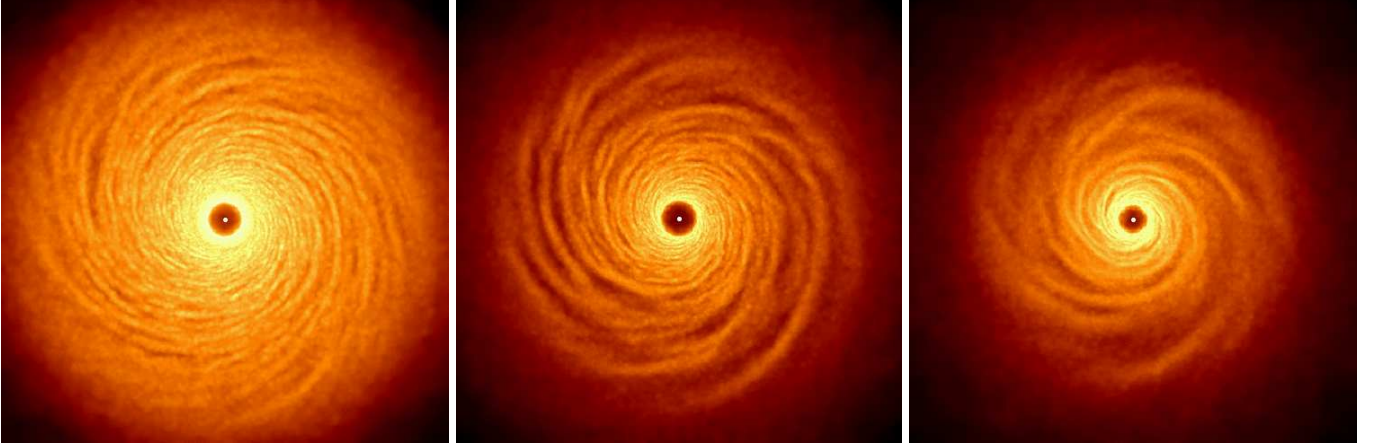
It has been suggested [6, 7] that, in a cold enough disc, where condition (2) is violated, gravitational instabilities would provide a source of energy dissipation, therefore heating up the disc and leading to a self-regulated state, with the disc close to marginal stability. Gravitationally induced spiral structures are also able to transport angular momentum. In this paper we examine the issue of characterizing the angular momentum transport and energy dissipation in self-gravitating discs. In particular, we would like to test the extent to which the transport due to gravitational instabilities can be described as a local, viscous process, amenable to a description analogous to Eq. (1), as proposed by Lin and Pringle [8].

Balbus and Papaloizou [9] have argued that the long-range nature of the gravitational field would preclude a local description of transport, as implicitly assumed in a viscous scenario. In particular, they have shown how the energy flux contains extra terms, related to wave transport, which are “anomalous” with respect to a viscous flux. Gammie [10] has performed local, shearing sheet, zero-thickness simulations of self-gravitating accretion discs, and has concluded that a local description is adequate in such “razor-thin” discs, extrapolating his results to discs for which  $H/r < 0.1$ . However, Gammie’s simulations are not appropriate to test global effects, since locality is set up from the beginning, and they are only valid for infinitesimally thin discs. Rice et al. [11, 12] have already shown using global, 3D simulations, how global effects can be important in the dynamics of self-gravitating discs, with respect to the related issue of fragmentation. Here we follow up the work of Rice et al., in order to quantify under which conditions a local model for transport in self-gravitating discs is justified.

## NUMERICAL SETUP

The simulations presented here were performed using smoothed particle hydrodynamics (SPH), a Lagrangian hydrodynamics code [see 13]. Our code uses a tree structure to calculate gravitational forces and the nearest neighbours of particles. The particles are advanced with individual time-steps [14], resulting in an enormous saving in computational times when a large range of dynamical timescales are involved.

We consider a system comprising a central star, modelled as a point mass  $M_*$ , surrounded by a disc with mass  $M_{disc}$ . We have performed several simulation using different values of the parameter  $q = M_{disc}/M_*$ . The initial surface density profile was taken to be



**Figure 1.** Equatorial density structure at the end of the simulations for (left)  $q = 0.05$ , (middle)  $q = 0.1$ , and (right)  $q = 0.25$ .

a power-law  $\Sigma \propto r^{-1}$ . The disc is initially gravitationally stable and the minimum value of  $Q$  is  $Q_{min} = 2$ . Our calculations are essentially scale-free. In dimensionless units, the disc extends from  $r_{in} = 0.25$  to  $r_{out} = 25$ . We have used  $N = 250000$  particles.

We use an adiabatic equation of state, with adiabatic index  $\gamma = 5/3$ . We explicitly solve the energy equation for our system, including heating from shocks and from artificial viscosity and introducing a cooling term, simply parameterized as:

$$\left( \frac{du_i}{dt} \right)_{cool} = -\frac{u_i}{t_{cool}}, \quad (3)$$

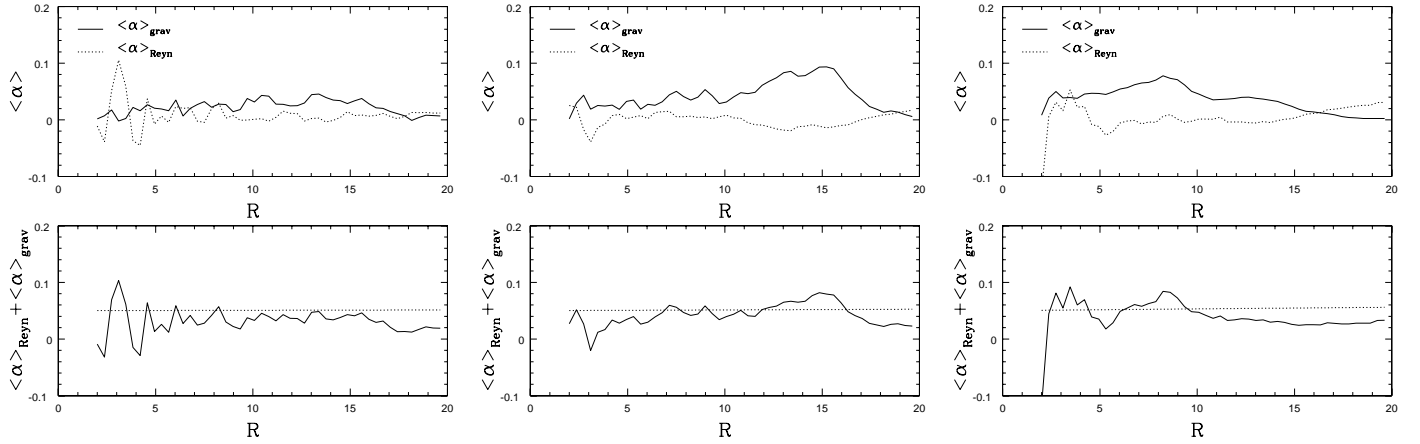
where  $u_i$  is the internal energy of a representative particle, and, as in Gammie [10] and in Rice et al. [12], the cooling time-scale  $t_{cool} = \beta \Omega^{-1}$ , where  $\beta$  is a free parameter, independent of radius. Gammie [10] and Rice et al. [12] have shown that if cooling is too fast, then the disc will fragment as a result of the gravitational instabilities. In our simulations we have used  $\beta = 7.5$ , in which case none of our discs was found to fragment.

## RESULTS

We have run our simulations for a few thermal timescales, in order to reach thermal equilibrium. In all cases the disc initially cools down until the value of  $Q$  becomes small enough for gravitational instabilities to become effective and to provide a source of heating to the disc. At later stages the disc settles into a self-regulated state with  $Q \approx 1$  over a large portion of the disc.

The  $r\phi$  component of the stress tensor (integrated in the  $z$ -direction) associated with self-gravity is given by [15]:

$$T_{r\phi}^{grav} = \int dz \frac{g_r g_\phi}{4\pi G}, \quad (4)$$



**Figure 2.** Effective  $\alpha$  produced by gravitational instabilities for (left)  $q = 0.05$ , (middle)  $q = 0.1$ , and (right)  $q = 0.25$ . The top panel shows the separate contribution of  $\alpha_{grav}$  and  $\alpha_{Reyn}$ , the lower panel shows the sum of the two contributions compared with the expected value from a local viscous model (dotted line).

where  $\mathbf{g}$  is the gravitational field of the disc. The hydrodynamic (Reynolds) contribution to the stress tensor is given by:

$$T_{r\phi}^{Reyn} = \Sigma \delta v_r \delta v_\phi, \quad (5)$$

where  $\delta \mathbf{v} = \mathbf{v} - \mathbf{u}$  is the velocity fluctuation,  $\mathbf{v}$  is the fluid velocity and  $\mathbf{u}$  is the mean fluid velocity.

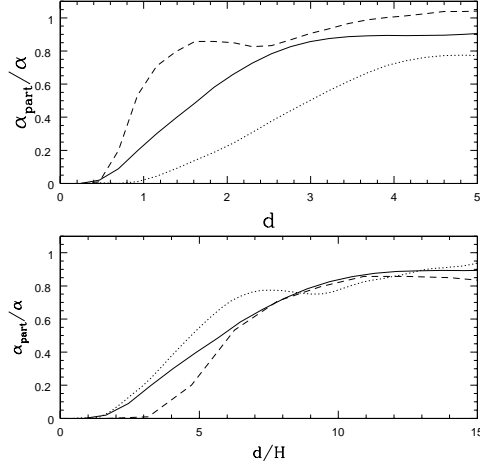
After averaging the stress tensor azimuthally and radially, over a small region  $\Delta r = 0.1r$ , we have computed the corresponding value of  $\alpha$  (see Eq. (1)):

$$\alpha(r) = \left| \frac{d \ln \Omega}{d \ln r} \right|^{-1} \frac{\langle T_{r\phi}^{grav} \rangle + \langle T_{r\phi}^{Reyn} \rangle}{\Sigma c_s^2}. \quad (6)$$

The resulting radial profiles of  $\alpha$  are shown in Fig. 2 for the three cases  $q = 0.05$ ,  $q = 0.1$ , and  $q = 0.25$ . The upper panels show separately the hydrodynamic and gravitational contributions to  $\alpha$ , while the bottom panels show the sum of the two. The results are also averaged in time, over the last 500 timesteps of the simulations.

The simple parameterization of the cooling time adopted here allows us to have a direct insight on the problem of characterizing transport properties and dissipation in the disc, by using the following result [16], which follows directly from the energy balance equation: if the disc is in thermal equilibrium, and *if heating is dominated by a purely viscous process*, described by Eq. (1), then the viscosity coefficient  $\alpha$  and the cooling time-scale  $t_{cool}$  satisfy the following relation:

$$\alpha = \left| \frac{d \ln \Omega}{d \ln r} \right|^{-2} \frac{1}{\gamma(\gamma-1)} \frac{1}{t_{cool} \Omega} \quad (7)$$



**Figure 3.** Contribution to the stress at  $r = 15$  from regions of the discs at a distance  $d$  from  $r$ . Dashed line:  $q = 0.05$ , solid line:  $q = 0.1$ . Dotted line:  $q = 0.25$

In our simulations  $t_{cool} = 7.5\Omega^{-1}$ , so from Eq. (7) we would expect that in thermal equilibrium, if energy dissipation can be described locally,  $\alpha$  should be approximately constant with radius (this is indicated with the dotted line in the bottom panels of Fig. 2). We can interpret the bottom panels of Fig. 2 in the following way: the solid line gives a measure of the actual torque induced by gravitational instabilities in our simulations, while the dotted line is the torque that a viscous process should exert in order to dissipate the amount of energy which is actually dissipated in the simulations. Therefore, when the computed value of  $\alpha$  is larger than the expected one, gravitational instabilities dissipate *less* energy than a viscous process, whereas in the opposite case, they dissipate *more* energy than a viscous process. Our results indicate that, up to a disc mass  $M_{disc} = 0.25M_*$ , gravitationally induced transport is fairly well described within a local framework. This result could also be anticipated, based on Fig. 1, that shows how the disc dynamics is dominated by high- $m$  modes, that dissipate on a short length-scale.

As a separate test for the locality of transport, we have also computed  $\alpha_{part}(r, d)$ , which is defined as the gravitational part of  $\alpha(r)$ , where the gravitational field  $\mathbf{g}$  is computed taking into account only those particles which are inside a spherical radius  $d$  from the radial point  $r$  at which we are computing the stress. This quantity gives us a measure of the size of the region that mostly contributes to the stress at a given point.

Fig. 3 shows the results for  $r = 15$  for the three simulations. The upper panel shows clearly that the more massive the disc is, the more far regions contribute to the stress. However, it should be noted that, since  $Q \approx 1$ , a more massive disc is also hotter, and therefore thicker. The bottom panel of Fig. 3 shows  $\alpha_{part}$  as a function of  $d/H$ , where  $H$  is the disc thickness. In all cases, more than 80% of the contribution to the gravitational torque come from a region with size  $\Delta \approx 10H$ . Transport is going to be local if  $\Delta \ll r$ . We can therefore conclude that, as long as  $M_{disc} < 0.25M_*$ , a disc with  $H/r \ll 0.1$  will be reasonably described in terms of a local model. This is basically the global analogous of the results obtained by Gammie [10] based on a local, 2D model.

Note that detailed vertical structure models of FU Orionis objects [5] lead to disc thickness  $H/r \approx 0.1$ , so that in this case we could expect global effects to be important in determining the emission properties of the disc, as suggested by Lodato and Bertin [17].

## CONCLUSIONS

We have performed global, three-dimensional SPH simulations of self-gravitating accretion discs. Our results show how the heating provided by gravitational instabilities can balance the cooling (that we have parameterized in a simplified way), leading to a self-regulated state where  $Q \approx 1$ . With a cooling rate  $t_{cool} = 7.5\Omega^{-1}$ , none of our simulations was found to fragment. We have characterized the transport properties of the disc, by computing the stress tensor associated with gravitational instabilities and comparing the corresponding viscous dissipation rate to the actual heating rate in the disc. We have found that, if the disc is less massive than  $0.25M_*$ , gravitational disturbances are dominated by high- $m$  modes, so that the transport properties of the disc can be described reasonably well in terms of a local, viscous formalism. In particular, we have found that the gravitational torque comes from a region of size  $\approx 10H$ , where  $H$  is the disc thickness. We can therefore expect that discs with an aspect ratio  $H/r < 0.1$  can be treated locally.

## ACKNOWLEDGMENTS

The simulations presented here were performed at the UK Astrophysical Fluids Facility (UKAFF).

## REFERENCES

1. Shakura, N., and Sunyaev, R., *Astronomy & Astrophysics*, **24**, 337 (1973).
2. Balbus, S., and Hawley, J., *Reviews of Modern Physics*, **70**, 1 (1998).
3. Toomre, A., *The Astrophysical Journal*, **139**, 1217 (1964).
4. Lodato, G., and Bertin, G., *Astronomy & Astrophysics*, **398**, 517 (2003).
5. Bell, K., and Lin, D., *The Astrophysical Journal*, **427**, 987 (1994).
6. Paczyński, B., *Acta Astronomica*, **28**, 91 (1978).
7. Bertin, G., and Lodato, G., *Astronomy & Astrophysics*, **350**, 694 (1999).
8. Lin, D., and Pringle, J., *Monthly Notices of the Royal Astronomical Society*, **225**, 607 (1987).
9. Balbus, S., and Papaloizou, J., *The Astrophysical Journal*, **521**, 650 (1999).
10. Gammie, C. F., *The Astrophysical Journal*, **553**, 174 (2001).
11. Rice, W. K. M., et al., *Monthly Notices of the Royal Astronomical Society*, **338**, 227 (2003).
12. Rice, W. K. M., et al., *Monthly Notices of the Royal Astronomical Society*, **339**, 1025 (2003).
13. Monaghan, J., *Annual Review of Astronomy & Astrophysics*, **30**, 543 (1992).
14. Bate, M., Bonnell, I., and Price, N., *Monthly Notices of the Royal Astronomical Society*, **277**, 362 (1995).
15. Lynden-Bell, D., and Kalnajs, A., *Monthly Notices of the Royal Astronomical Society*, **157**, 1 (1972).
16. Pringle, J., *Annual Review of Astronomy & Astrophysics*, **19**, 137 (1981).
17. Lodato, G., and Bertin, G., *Astronomy & Astrophysics*, **375**, 455 (2001).